Chiral Fermion Pair Production from Parallel $\vec{E}$ and $\vec{B}$ Fields (and the Physics Involved)– Gerald B. Cleaver

As recent articles [1–7] further suggest, matter/antimatter (MAM)-based propulsion systems may be viable options for both intrasolar system and interstellar travel. Several critical design and engineering issues, especially with regard to the interstellar case, have been raised in these articles and will likely take a long time to resolve. As the articles have discussed, MAM could be (i) collected in advance of a space journey (and then stored in separate magnetic bottles or rings until needed for thrust) and/or collected along the journey through interstellar space [2–5] , or (ii) created on-board the ship [1]. One suggestion for advanced collection (or for sporadic collection en route during encounters with solar systems) is from gaseous Jovian-like planets or (as suggested by PAMELA results) from the Van Allen belt of earth-like planets. Alternately, Richard Obousy investigates in [1] the feasibility and functionality of on-board MAM creation via Schwinger pair production from the vacuum of space through intense electric field quantum effects. With regard to this, Richard notes that lasars are reaching the critical strength to produce in this manner real electron-positron pairs. For MAM collection or creation, additional external magnetic fields are then needed to align the MAM in the chosen direction of thrust. Collection and creation options were considered and compared in [2], including pair production rates and related costs.

For maximal probability of a successful intrasolar/interstellar voyage, a spacecraft should have multiple redundancies and back-ups of all of its critical systems, including of its propulsion system. Thus, I can image the ideal spacecraft using MAN propulsion would be designed with the equipment to both collect and create MAM, with creation especially taking over as an emergency option if stored MAM leaks out of its magnetic containment chambers or is annihilated prematurely within the magnetic chambers by matter leaking in. With this in mind, I'd like to mention an additional (non-laser) method for Schwinger pair production that calls on weak electric fields $\vec{E}$ aligned in parallel with magnetic fields $\vec{B}$, $\vec{E}||\vec{B}$. (I want to acknowledge that this pair production discussion is based on unpublished class notes of John Preskill (Physics Department, Caltech) from the latter 1980's/early 1990's. The figures below also come from Preskill's notes.)

As I will discuss in more detail in some upcoming blogs, pair production from parallel electric and magnetic fields is related to chiral symmetry breaking (CSB) that occurs for left- and right-handed elementary particles (specifically for quarks) in the strong coupling (low energy) limit of QCD. (Essentially CSB means that global complex phases that could be picked up by left-handed particles and their right-handed counterparts are no longer independent, but become correlated.) CSB allows an interaction term $F_{\mu\nu}\tilde{F}^{\mu\nu}$ between the field strength tensor $F_{\mu\nu}$ and its dual field strength tensor $\tilde{F}^{\rho\sigma} \equiv \epsilon_{\rho\sigma\mu\nu}F_{\mu\nu}$. For electromagnetics, this term is simply $F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$. (The dot product indicates that only the parallel components of $\vec{E}$ and $\vec{B}$ interact).

Why this term can result in particle/antiparticle pair production is interesting. Let's look at it from the hamiltonian (energy) approach. To start, consider spin $S = \frac{1}{2}$ fermions of mass m and electric charge e in a constant magnetic field $\vec{B}$ aligned along the z-axis, $\vec{B} = B\hat{z}$. The electromagnetic gauge field $\vec{A}$ producing the physical magnetic field $\vec{B}$ can be chosen as $\vec{A} = Bx\hat{y}$. The square of the hamiltonian of a fermion in this field is

$$\begin{aligned} H^2 &= (\vec{p} - e\vec{A})^2 + m^2 - ge\vec{B}\cdot\vec{S} \\ &= p_z^2 + p_x^2 + (p_y^2 - eBx)^2 + m^2 - geBS_z. \end{aligned}$$

with $\vec{p}$ the fermion's momentum operator, and g its gyromagnetic ratio. g will be very close to 2 (and so we will set it to 2). Momenta p_z and p_y are constants of motion since they commute with the hamiltonian. The contributions

$$p_x^2 + (eB)^2(x - x_o)^2$$

have the form of a harmonic oscillator and therefore have the spectrum $(2n+1)eB$, where n is an integer. Thus,

$$H^2 = p_z^2 + (2n+1)eB - eB(2S_z) + m^2,$$

where $2S_z = \pm 1$.

We see that in the limit of massless fermions ($m^2 = 0$), there is a zero energy mode for $n = 0$ and $2S_z = +1$. Then parallel electric and magnetic fields can excite the zero energy mode, resulting in pair production: Starting at the zero-energy mode (and ignoring the higher energy modes) we now turn on a weak electric field $\vec{E} = E\hat{z}$ parallel to $\vec{B}$. The component of the gauge field responsible for $\vec{E}$ is $A_z = Et$ (with gauge choice of $A_0 = 0$). For small E (equivalently, for slow changes in A_z) the energy level shift's adiabatically and leaves $n = 0$ and $S_z = \frac{1}{2}$ unchanged. The shifted zero-mode hamiltonian reduces to

$$H^2 = (p_z - Et)^2 + m^2.$$

Choosing a finite level cut-off, the energy levels are discrete and, with increasing time t, they move along a mass-shell hyperbola like beads on a neckless.

Each energy mode in Figure 1 can be assigned a helicity. All have $S_z = \frac{1}{2}$ and the energy mode will be moving to the left or right depending on whether its group velocity is negative or positive. Unoccupied positive energy modes with positive p_z are right handed fermion states, while those with negative p_z are left-handed. The opposite is true for the filled negative energy modes.

Using the Dirac sea concept, for the ground state of the system we assign all of the negative energy states to be filled and all of the positive energy states to be empty (see Figure 1). Even for a weak electric field of minimum energy, the negative energy fermions will have enough energy to "jump" across the $2m$ energy gap separating the negative and positive energy modes, excited by the time varying gauge field A_z. The physical realization of this is chiral particle pair production: a right-handed particle (filled positive energy state) and a left-handed antiparticle (empty negative energy state, that is, a "hole") (see Figure 2). Both the net linear momentum and the net angular momentum of each pair will be zero. In the $m = 0$ limit, an arbitrarily weak electric field can induce chiral pair production. Hyperbolic arcs become crossed lines, with modes not changing direction when crossing $p_z = 0$ (see Figure 3).

Thus, we get a glimpse of the intriguing quantum physics of chiral pair production via parallel $\vec{E}$ and $\vec{B}$ fields. While comparison of design feasibility of $\vec{E}||\vec{B}$ systems for MAM generation in interstellar spacecraft with that of high energy laser devices will be reserved for future blogs, it is hoped this introduction will promote further contemplation of $\vec{E}||\vec{B}$ system viability.

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[1] Richard Obousy, Vacuum to Antimatter-Rocket Interstellar Exploration System (VARIES): A Proposed Program for an Interstellar Rendezvous and Return Architecure.

[2] P. Gilster, Antimatter: Finding the Fuel, www.centauri-dreams,org

[3] R. Keane and W.M. Zhang, Beamed Core Antimatter Propulsion: Engine Design and Optimisation, arXiv/1205.2281.

[4] Antimatter Propulsion Engine Redesigned Using CERN's Particle Physics Simulation Toolkit,

reporting on [3], www.technologyreview.com/blog/arxiv/27847.

[5] Jennifer Oullette, Revving Up the Antimatter Engine, Discovery News, May 16, 2012, reporting on [3].

[6] P. Gilster, Antimatter: The Production Problem, www.centauri-dreams,org

[7] P. Gilster, Antimatter: Toward a Beam Core Drive, www.centauri-dreams,org

Figure 1.

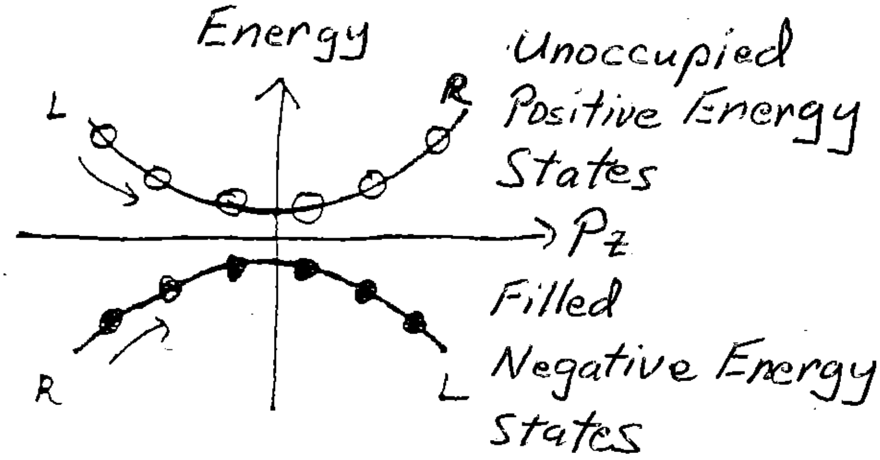

Figure 2.

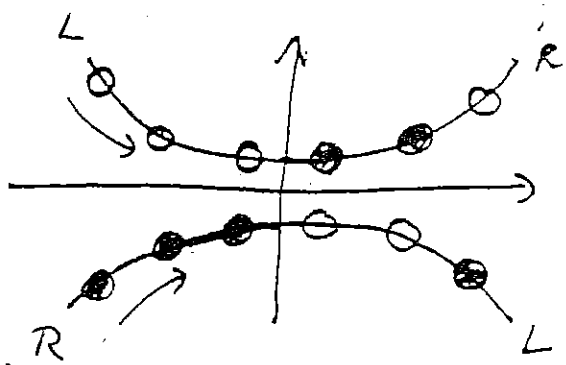

Figure 3.