Two-Burn Escape Maneuver for High ΔV Capable Spacecraft

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Presentation Objectives

- Reintroduce the Two-Burn Escape maneuver
- Discuss underlying physics and demonstrate proof
- Show opportunities for application in near term missions

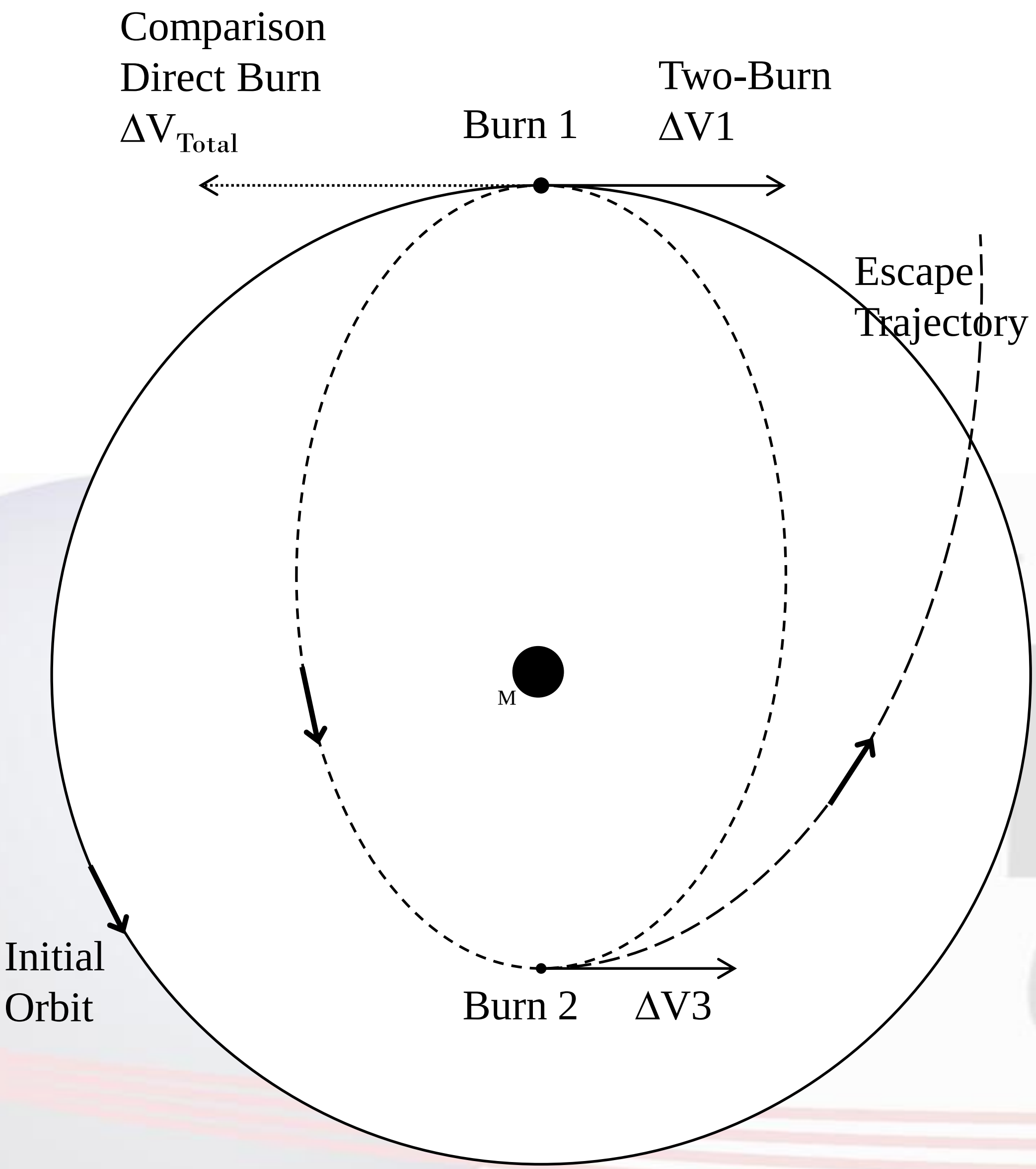
Genesis

- In most low thrust derivations the idea that escape velocity is best achieved by accelerating along the velocity vector
  - Reason given is that change in specific orbital energy is a function of velocity and acceleration

$$\frac{dE}{dt} = \vec{V} \cdot \vec{a}$$

  - However Levin, 1952 suggested that while this is a locally optimal solution it might not be a globally optimal one.
  - Turning acceleration inward would drop periapse giving a higher velocity later in the trajectory. Acceleration at that point would be dotted against a higher magnitude V giving a greater rate of change of mechanical energy

Derivation


Comparison
Direct Burn
ΔV_{Total}
Burn 1
Two-Burn
$\Delta V1$
Escape
Trajectory
M
Initial
Orbit
Burn 2
$\Delta V3$

Derivation

- Consider the derivation for the specific orbital energy for the maneuver. Position 3 is the status of the vehicle after the burn at periapse

$$\xi_3 = \frac{(V_2 + \Delta V_3)^2}{2} - \frac{\mu}{r_3}$$

- Position 2 is the status of the vehicle before the burn. Orbital radius at position 2 and 3 are equal as the burn is considered impulsive

$$r_2 = 2a_1 - r_1$$

$$V_2 = \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a_2}}$$

- Position 1 is the status of the vehicle after the burn from the initial orbit. Radius is same as at position 0, initial position of the vehicle

$$a_1 = -\frac{\mu}{2\xi_1}$$

$$\xi_1 = \frac{(V_0 + \Delta V_1)^2}{2} - \frac{\mu}{r_1}$$

Derivation

- Combine all of the above to get

$$\xi_3 = \frac{\left(\sqrt{\dfrac{-2\mu}{\dfrac{\mu}{\sqrt{\dfrac{V_1^{\,2}}{2} - \dfrac{\mu}{r_1}}} + r_1} + 2\sqrt{\dfrac{V_1^{\,2}}{2} - \dfrac{\mu}{r_1}}} + \Delta V_3 \right)^2}{2} + \frac{\mu}{\dfrac{\mu}{\sqrt{\dfrac{V_1^{\,2}}{2} - \dfrac{\mu}{r_1}}} + r_1}$$

- Some simple algebra gives

$$\xi_3 = \frac{\left(\sqrt{\dfrac{4\mu^2 + r_1^{\,2} V_1^{\,4} - 4\mu r_1 V_1^{\,2}}{r_1^{\,2} V_1^{\,2}}} + \Delta V_3 \right)^2}{2} + \frac{\mu r_1 V_1 - 2\mu^2}{r_1^{\,2} V_1^{\,2}}$$

Derivation

- Noting that the initial orbital velocity is

$$V_0 = \sqrt{\frac{\mu}{r_0}}$$

- Then including it and the first ΔV maneuver yields

$$\xi_3 = \frac{\left(\sqrt{\frac{4\mu^2 + r_0^{\ 2}\left(\sqrt{\frac{\mu}{r_0}} - \Delta V_1\right)^4 - 4\mu r_0\left(\sqrt{\frac{\mu}{r_0}} - \Delta V_1\right)^2}{r_0^{\ 2}\left(\sqrt{\frac{\mu}{r_0}} - \Delta V_1\right)^2}} + \Delta V_3 \right)^2}{2} + \frac{\mu r_0\left(\sqrt{\frac{\mu}{r_0}} - \Delta V_1\right) - 2\mu^2}{r_0^{\ 2}\left(\sqrt{\frac{\mu}{r_0}} - \Delta V_1\right)^2}$$

Derivation

- Noting that

$$V_1 = \sqrt{\frac{\mu}{r_0}} - \Delta V_1$$

- After considerable algebraic reduction the equation for specific mechanical energy becomes

$$\xi_3 = \left(V_1^{\,2} - 2\frac{\mu}{r_0}\right)\cdot\left(\frac{1}{2} - \frac{\Delta V_3}{V_1}\right) + \frac{\Delta V_3^{\,2}}{2}$$

- For a direct burn out of the initial orbit the final specific mechanical energy becomes

$$\xi_3 = \frac{\left(\sqrt{\frac{\mu}{r_0}} + \Delta V_1 + \Delta V_3\right)^2}{2} - \frac{\mu}{r_0}$$

Derivation

- Setting the two equal gives

$$\frac{\left(\sqrt{\frac{\mu}{r_0}}+\Delta V_1+\Delta V_3\right)^2}{2}-\frac{\mu}{r_0}=\left(V_1^{\ 2}-2\frac{\mu}{r_0}\right)\cdot\left(\frac{1}{2}-\frac{\Delta V_3}{V_1}\right)+\frac{\Delta V_3^{\ 2}}{2}$$

- Solving for the squared term and squaring yields

$$V_1^{\ 2}-2V_1\Delta V_3+\frac{4\mu\Delta V_3}{V_1 r_0}=\frac{\mu}{r_0}+2\Delta V_1\sqrt{\frac{\mu}{r_0}}+\Delta V_1^{\ 2}+2\Delta V_1\Delta V_3$$

- After substituting for V_1 and going through considerable algebra the equation becomes

$$\Delta V_1+\Delta V_3=\sqrt{\frac{\mu}{r_0}}$$

- Thus this new maneuver outperforms a direct burn when the overall ΔV budget exceeds the initial orbital velocity

Analogy

- Consider a car on the interstate with a rocket in the trunk.
  - The rocket gives a set constant acceleration
  - Starting at zero turn the rocket on until it reaches 10 mph
  - The change in kinetic energy for the car (assuming negligible propellant mass used) is

$$\Delta KE = \frac{V_f^{\,2}}{2} - \frac{V_i^{2}}{2} = \frac{10^2}{2} - \frac{0^2}{2} = 50mph^2$$

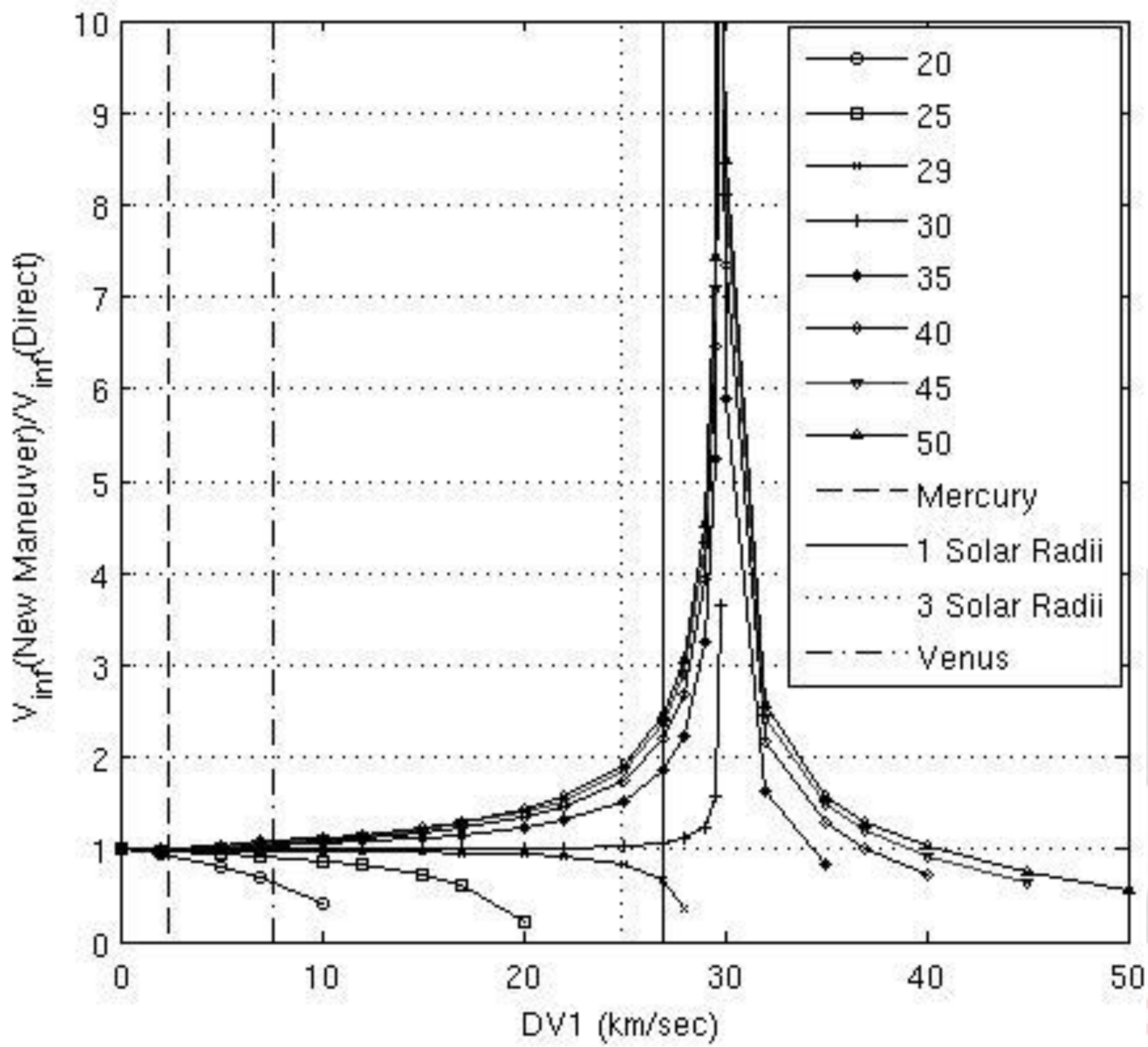
  - Consider the same car moving at 50 mph that accelerates to 60 mph

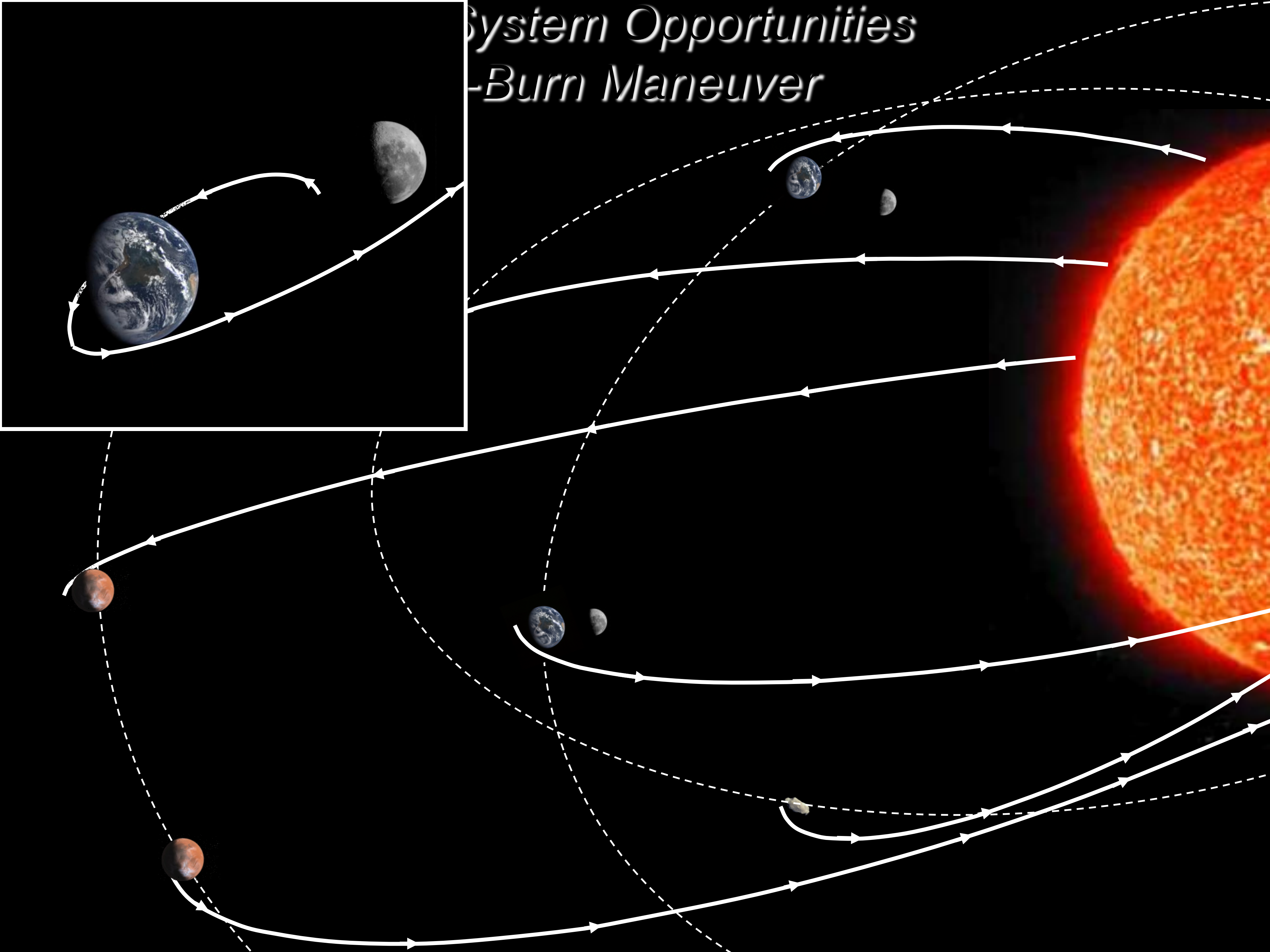
$$\Delta KE = \frac{V_f^{\,2}}{2} - \frac{V_i^{2}}{2} = \frac{60^2}{2} - \frac{50^2}{2} = 55mph^2$$

  - Where did the extra energy come from? The propellant used had a greater total energy. It had the same latent energy as the car at 0 mph but also had the initial kinetic energy that comes from moving at 50 mph

Potential

- This maneuver can dramatically increase achievable escape velocity
- Applies to deep space and inner solar system exploration

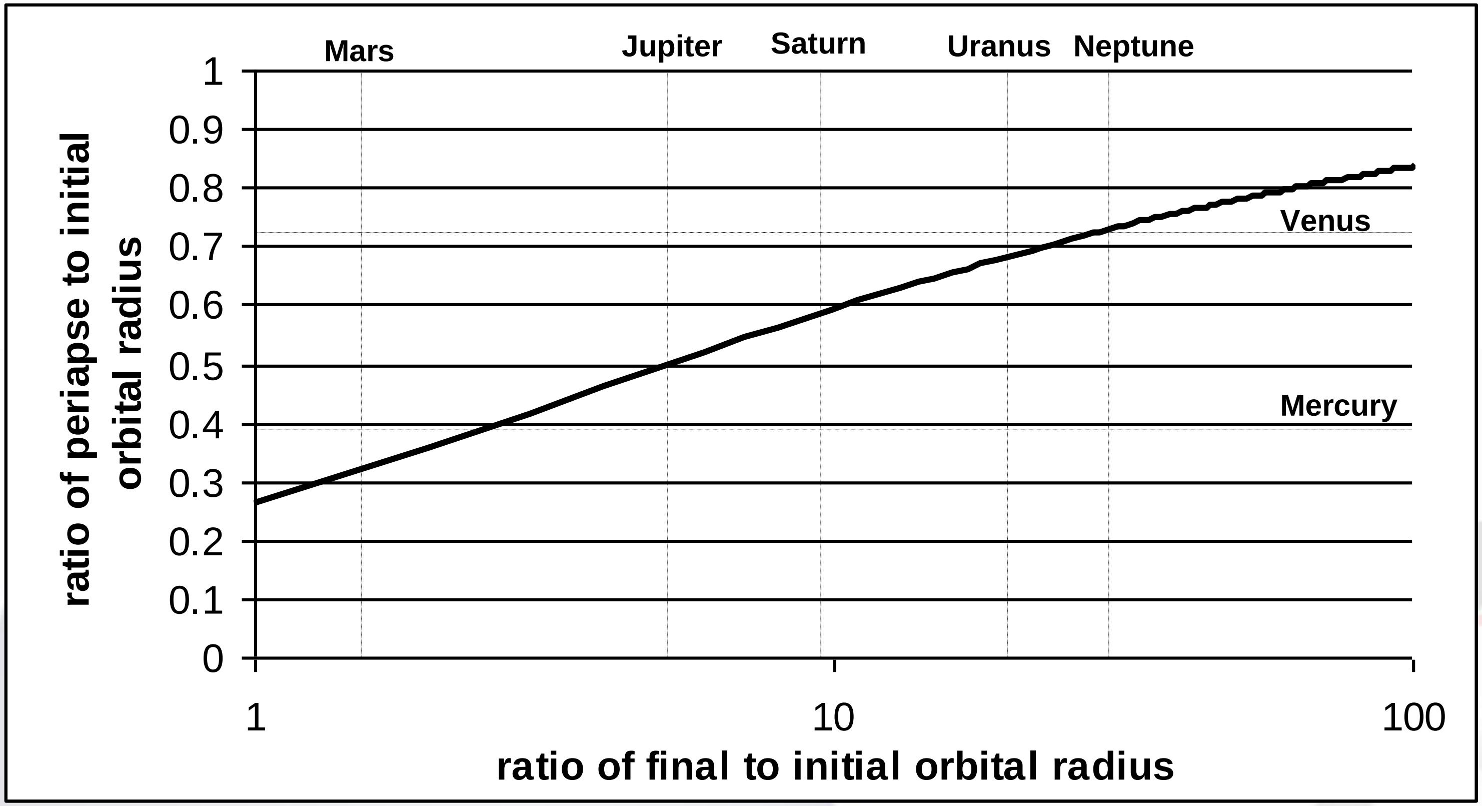




ystem Opportunities
-Burn Maneuver

Potential

- Using the two burn maneuver can raise orbit much faster than Hohmann transfers



Note that the dotted lines show ratios relative to Earth

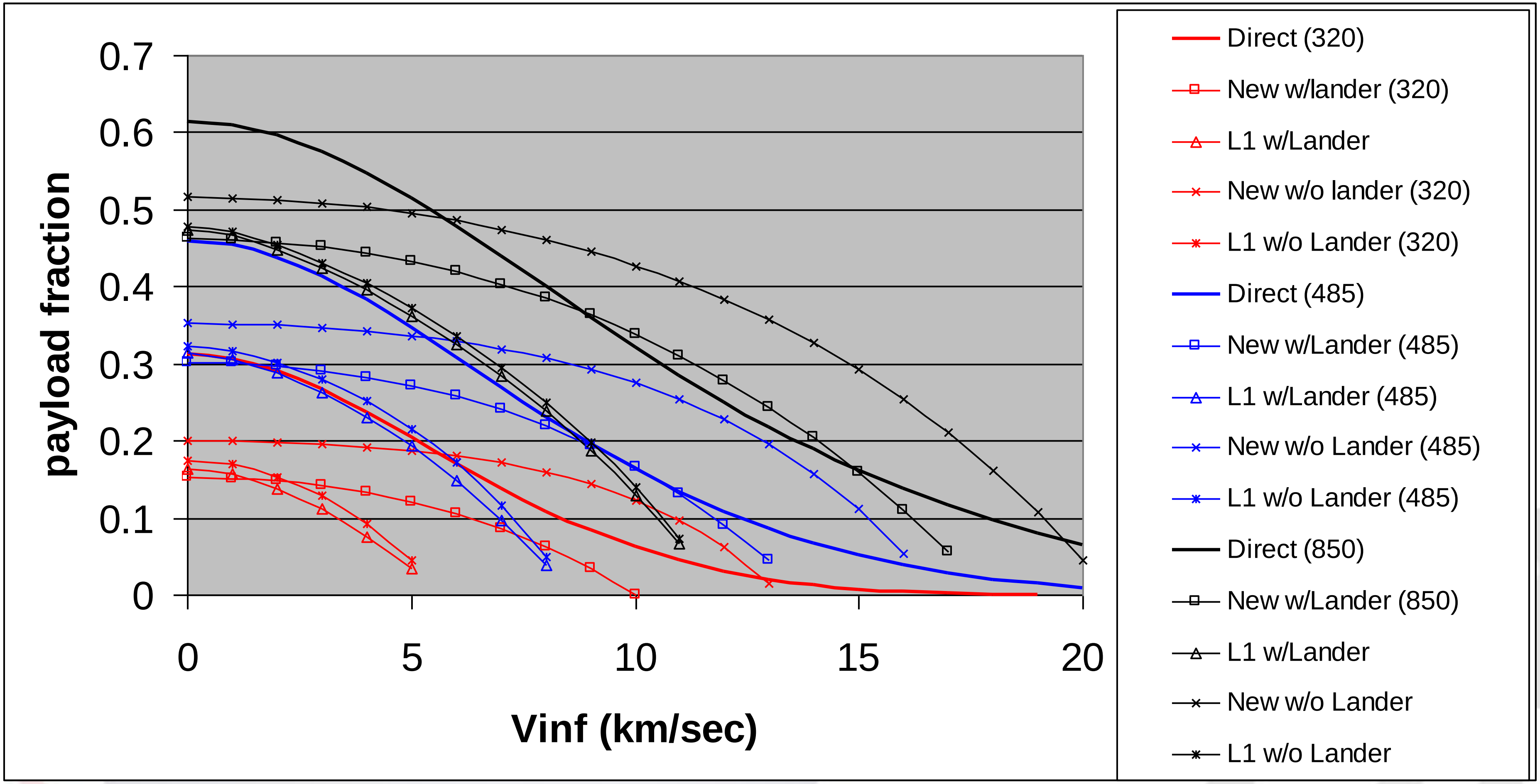
L1 Rendezvous

- Compare a mission that conducts a direct burn to escape to one that parks at L1, docks with fuel raised from the moon, and conducts the new maneuver to escape
  - Direct option
    - Two stage vehicle that delivers a payload to a given V_∞. Both stages burn instantaneously and one immediately after the other
  - new option w/lander
    - First option is that mass in LEO contains same payload as above, a stage to insert said payload to L1 and a separate stage that contains a lander and propellant to land on the moon. Lander is sized to carry enough fuel on launch from moon to rendezvous with payload at L1 and execute new maneuver
  - L1 option w/lander
    - Same as above, but burn is conducted to take spacecraft directly out of L1 to escape
  - new option w/o lander
    - Same as new option, but Hohmann transfer stage carries payload and empty stage for escape maneuver. Lander is considered part of infrastructure and will carry full tank up to mate with payload, and carry empty stage down for next maneuver
  - L1 option w/o lander
    - Same as above but direct burn to escape

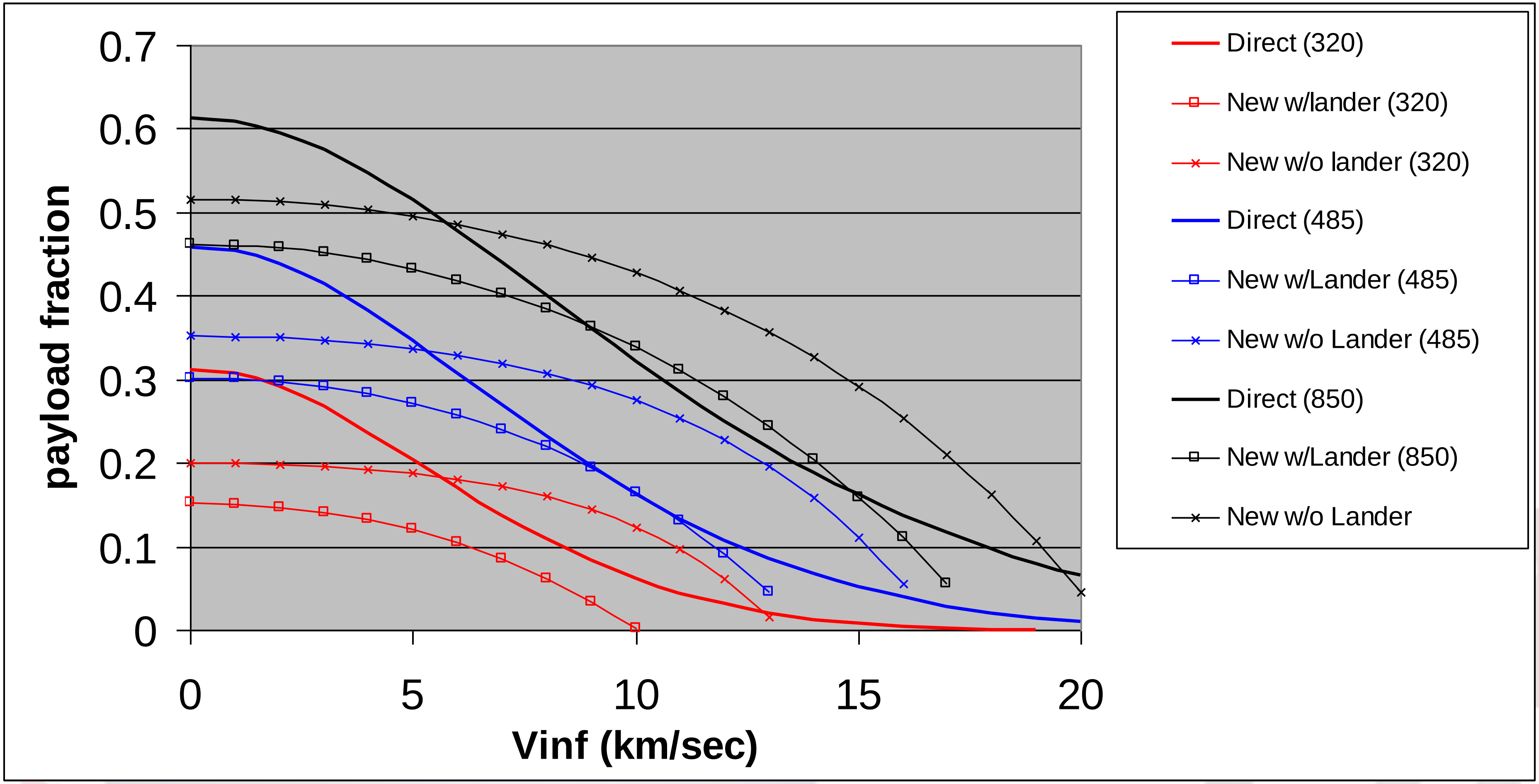
L1 Rendezvous

- Assumptions/Calculations
  - Three propulsive options considered
    - 320 sec Isp, 0.1 inert mass fraction (GOx/GH2, UMDH/N2O4)
    - 485 sec Isp, 0.12 inert mass fraction (Lox/LH2)
    - 850 sec Isp, 0.2 inert mass fraction (Nuclear thermal)
  - Hohmann transfer to L1
    - 3.082 km/sec first burn (also used for TLI burn)
    - 0.828 km/sec second burn
  - Lunar operations
    - 2.376 km/sec launch from Moon
    - 0.89+2.03 km/sec land on Moon
  - Other
    - No additional mass accounted for lander structure
    - Hohmann transfers are assumed to be impulsive
    - Launch DV does not account for gravitational losses or Lunar rotation
  - Options Considered but not shown here
    - All mass to lunar surface, new maneuver after launch from moon
    - HOH stage carries payload and lander to L1; payload and lander break apart there

L1 Rendezvous



L1 Rendezvous



L1 Rendezvous

- Discussion
  - The maneuver offers a lot of advantage for missions with V_∞ in the 5 to 20 km/sec range
    - The new maneuver can double the V_∞ achieved for a given payload ratio in this velocity range
  - The V_∞ range is within that for many missions of interest
    - Short term stay missions to Mars require V_∞ of 5-6 km/sec
    - Hohmann transfer to Jupiter requires V_∞ of 8-9 km/sec
    - Pluto-Kuiper Express launched in 2006 achieved a V_∞ of 16.21 km/sec
  - A similar advantage may exist for an Earth-Sun L1 rendezvous given propellant mined from an NEO; I haven't investigated that yet
  - Particular missions
    - It may be that the best use of the new maneuver for round trip missions to Mars is to lower trip time. The flat nature of the curve suggests a gain in V_∞ from the usual 5-6 km/sec range can be had for a moderate reduction in payload fraction
    - This maneuver combined with a solar new maneuver could be powerful for solar escape missions, Pluto-Kuiper-Oort cloud missions and orbit raising to the gas giants

Targets

- Earth-Sun Lagrange points
  - Invaluable for scientific research, early warning and communications for crewed deep space missions
- Mars
  - Iconic target for next step in human exploration
- NEO's
  - Rich source of resources, could be leveraged for Mars exploration, possible intermediate step to Mars
- Asteroid belt
  - Additional source for resources, logical next step after Mars
- Outer Solar system and interstellar
  - Multiple scientific targets of interest, capture public attention with deep space robotic probes

Unanswered Questions

- Semi-Tangential solutions need deeper exploration
  - Involve three body scenario
  - Gravity assist potentially has strong synergy with maneuver
    - VEEGA
    - Jupiter swingby
- Survivability of close approaches
  - To sun
    - Solar probe B designed to survive approach within 3 solar radii
  - To Earth
    - Trade perigee vs. atmospheric drag
    - Solar min vs. solar max, nighttime vs. daytime
- Synergy with near term technologies
  - Nuclear Thermal Propulsion
  - Aeroassist
- Desirable additions to a lunar architecture
  - Lander options
  - ISRU facilities at lunar base
  - On orbit refueling options